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Solution.—Let AB be the side of any given square, upon which describe a semicircle, and on each radius AC and CB , describe other semicircles as shown in the diagram.

From B as centre and radius BC describe the arc Cg . Join Ag cutting the radius CD in E . Then, assuming $AB=1$, we have by similar triangles $Ag^2 : AB^2 :: AC^2 : AE^2$ or $1 - \frac{1}{4} : 1 :: \frac{1}{4} : \frac{1}{5}$. Hence the square described on $AE = \frac{1}{5}$ the square on AB .

Again, from A as centre and radius AC describe the arc Ca' — Aa' being perpendicular to AB . Join Ba' cutting the semicircle on CB in a . Then because $Ba'^2 : AB^2 :: CB^2 : Ba^2$, therefore $1 + \frac{1}{4} : 1 :: \frac{1}{4} : \frac{1}{5}$; or the square described on $Ba = \frac{1}{5}$ the square on AB .

In a similar manner, $Bb = \frac{1}{6}$, $Bc = \frac{1}{7}$, $Bd = \frac{1}{8}$, $Am = \frac{1}{9}$, $An = \frac{1}{10}$, $Ao = \frac{1}{11}$, $Ap = \frac{1}{12}$ &c., of the square on AB ; and there is a regular law governing the construction as shown in the diagram.

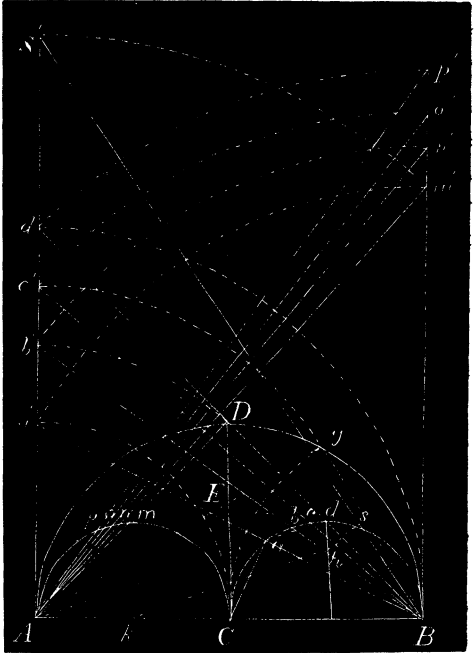
Again, suppose we wanted to construct a square $= \frac{7}{13}$ of a given square. From the above construction we know that $Bs^2 = \frac{1}{13}$ of AB^2 and $Bc^2 = \frac{1}{7}$ of AB^2 . With radius Bs describe the arc sh cutting Bc in h . Join Ac and draw through h a line parallel with Ac cutting AB in k ; then $Bk^2 = \frac{7}{13}$ of AB^2 . Because by similar triangles $Be^2 : BA^2 :: Bh^2 : Bk^2$, or, multiplying the antecedents by 7, $7Be^2 : BA^2 :: 7Bh^2 : Bk^2$. But by construction $7Be^2 = BA^2$; $\therefore 7Bh^2 = Bk^2$, and because $Bh^2 = \frac{1}{13}$ of BA^2 $\therefore Bk^2 = \frac{7}{13}$ of BA^2 , and similarly for any other fractional part.

It is evident that a circle, or any regular figure, may be divided in similar fractional parts by the same construction.

SOLUTION OF A PROBLEM.

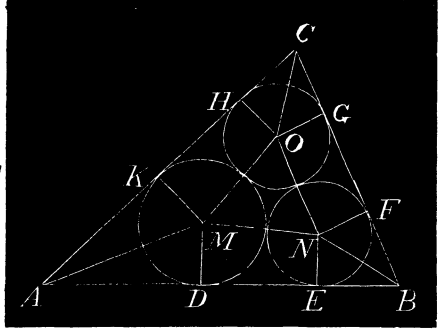
BY E. B. SEITZ, GREENVILLE, OHIO.

To determine the radii of three circles inscribed in a triangle whose sides are a, b, c , each cir. touching the other two, and also two sides of the triangle.



Solution.—Let ABC be the triangle, M, N, O , the centers of the circles, D, E, F, G, H, K , the points of tangency.

Put $MD = x$, $NE = r_1^2 x$, $OG = r_2^2 x$, and let r = the radius of the inscribed circle of the triangle. Then $AD = AK = x \cot \frac{1}{2}A$, $BE = BF = r_1^2 x \cot \frac{1}{2}B$, $CG = CH = r_2^2 x \cot \frac{1}{2}C$, $DE = 2r_1 x$, $HK = 2r_2 x$, $FG = 2r_1 r_2 x$, and we obtain the following equations.



$$x \cot \frac{1}{2}A + 2r_1 x + r_1^2 x \cot \frac{1}{2}B = c, \dots \dots \dots (1)$$

$$x \cot \frac{1}{2}A + 2r_2 x + r_2^2 x \cot \frac{1}{2}C = b, \dots \dots \dots (2)$$

$$r_1^2 x \cot \frac{1}{2}B + 2r_1 r_2 x + r_2^2 x \cot \frac{1}{2}C = a. \dots \dots \dots (3)$$

By Trigonometry we have $b(\cot \frac{1}{2}A - \tan \frac{1}{2}B) = c(\cot \frac{1}{2}A - \tan \frac{1}{2}C)$, . . . (4)

$$ar_1^2(\cot \frac{1}{2}B - \tan \frac{1}{2}A) = cr_1^2(\cot \frac{1}{2}B - \tan \frac{1}{2}C), \dots \dots \dots (5)$$

$$\frac{\sin \frac{1}{2}B \cos \frac{1}{2}B}{b} = \frac{\sin \frac{1}{2}C \cos \frac{1}{2}C}{c}, \dots (6) \quad \frac{\sin \frac{1}{2}A \cos \frac{1}{2}A}{a} = \frac{\sin \frac{1}{2}C \cos \frac{1}{2}C}{c} \dots (7)$$

Dividing (1) by (2) and (3), and clearing of fractions, we have

$$b(\cot \frac{1}{2}A + 2r_1 + r_1^2 \cot \frac{1}{2}B) = c(\cot \frac{1}{2}A + 2r_2 + r_2^2 \cot \frac{1}{2}C), \dots (8)$$

$$a(\cot \frac{1}{2}A + 2r_1 + r_1^2 \cot \frac{1}{2}B) = c(r_1^2 \cot \frac{1}{2}B + 2r_1 r_2 + r_2^2 \cot \frac{1}{2}C). (9)$$

Subtracting (4) from (8) and (5) from (9), we have

$$b(\tan \frac{1}{2}B + 2r_1 + r_1^2 \cot \frac{1}{2}B) = c(\tan \frac{1}{2}C + 2r_2 + r_2^2 \cot \frac{1}{2}C), \dots (10)$$

$$a(\cot \frac{1}{2}A + 2r_1 + r_1^2 \tan \frac{1}{2}A) = c(r_1^2 \tan \frac{1}{2}C + 2r_1 r_2 + r_2^2 \cot \frac{1}{2}C). (11)$$

Multiplying (10) by (6) and (11) by (7), and extracting the square root, we have

$$\sin \frac{1}{2}B + r_1 \cos \frac{1}{2}B = \sin \frac{1}{2}C + r_2 \cos \frac{1}{2}C, \dots \dots \dots (12)$$

$$\cos \frac{1}{2}A + r_1 \sin \frac{1}{2}A = r_1 \sin \frac{1}{2}C + r_2 \cos \frac{1}{2}C. \dots \dots \dots (13)$$

Subtracting (13) from (12), we find

$$\begin{aligned} r_1 &= \frac{\sin \frac{1}{2}C + \cos \frac{1}{2}A - \sin \frac{1}{2}B}{\sin \frac{1}{2}C - \sin \frac{1}{2}A + \cos \frac{1}{2}B} = \frac{\sin \frac{1}{2}C + \sin \frac{1}{4}C \cos \frac{1}{4}(2B + C)}{\sin \frac{1}{2}C + \sin \frac{1}{4}C \cos \frac{1}{4}(2A + C)} \\ &= \frac{\cos \frac{1}{4}C + \cos \frac{1}{4}(2B + C)}{\cos \frac{1}{4}C + \cos \frac{1}{4}(2A + C)} = \frac{\cos \frac{1}{4}B \cos \frac{1}{4}(\pi - A)}{\cos \frac{1}{4}A \cos \frac{1}{4}(\pi - B)} = \frac{1 + \tan \frac{1}{4}A}{1 + \tan \frac{1}{4}B}. \end{aligned}$$

Similarly we find

$$r_2 = \frac{\cos \frac{1}{4}C \cos \frac{1}{4}(\pi - A)}{\cos \frac{1}{4}A \cos \frac{1}{4}(\pi - C)} = \frac{1 + \tan \frac{1}{4}A}{1 + \tan \frac{1}{4}C}.$$

$$\begin{aligned}
 \text{From (3) we have } x &= \frac{a}{r_1^2 \cot \frac{1}{2}B + 2r_1 r_2 + r_2^2 \cot \frac{1}{2}C} \\
 &= \frac{a \sin \frac{1}{2}B \sin \frac{1}{2}C}{r_1^2 \cos \frac{1}{2}B \sin \frac{1}{2}C + 2r_1 r_2 \sin \frac{1}{2}B \sin \frac{1}{2}C + r_2^2 \sin \frac{1}{2}B \cos \frac{1}{2}C} \\
 &= \frac{r \cos \frac{1}{4}A \cos \frac{1}{4}(\pi - B) \cos \frac{1}{4}(\pi - C)}{2 \cos \frac{1}{4}\pi \cos \frac{1}{4}B \cos \frac{1}{4}C \cos \frac{1}{4}(\pi - A)} = \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}B)(1 + \tan \frac{1}{4}C)}{1 + \tan \frac{1}{4}A} \\
 \therefore r_1^2 x &= \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}A)(1 + \tan \frac{1}{4}C)}{1 + \tan \frac{1}{4}B}, r_2^2 x = \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}A)(1 + \tan \frac{1}{4}B)}{1 + \tan \frac{1}{4}C}.
 \end{aligned}$$

SOLUTION OF MR. CHURCH'S PROBLEM.

BY PROF. E. W. HYDE, ITHACA, N. Y.

"Given four points [no one point lying within the triangle formed by the other three] to construct geometrically the axis and focus of the parabola passing through them.

Solution.—Let the four points be a^o, b, c, d^o . Regard them as lying in the horizontal plane of projection, and draw a ground line GL through two of them as b and c . Take some point p [marked $p^h p^v$] in the second angle, and draw the right lines pd and pa piercing the vertical plane in d_1 and a_1 . b and c are in GL and therefore in both the horizontal and vertical planes of projection.

Draw in the vertical plane a horizontal line L^o through p^o . Now if an ellipse be drawn passing through the four points a^o_1, b, c , and d^o_1 , and also tangent to the line L^o , and if p be taken as the vertex of the projecting cone, the ellipse $a^o_1 b c d^o_1 L^o$ will be projected into the required parabola, upon the horizontal plane. For by the construction b and c are their own projections, a^o_1 is projected into a^o , and d^o_1 into d^o , and L^o is projected to infinity. It follows therefore that $p^h t^h$ is a diameter of the parabola. It is not necessary to construct the ellipse. Draw by Pascal's theorem a tangent to the ellipse at

